From
APOLLONIUS OF PERGA
CONICS
BOOKS ONE - SEVEN

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[Proposition] 13

If a cone is cut by a plane through its axis, and is also cut by another plane which on the one hand meets both [lateral] sides of the axial triangle, and on the other hand, when continued, is neither parallel to the base [of the cone] nor antiparallel to it, and if the plane of the base of the cone and the cutting plane meet in a straight line perpendicular either to the base of the axial triangle or to it continued, then any [straight] line drawn parallel to the common section of the [base and cutting] planes from the section of the cone to the diameter of the section will be equal in square to some [rectangular] plane applied to a straight line to which the diameter of the section is as the square on the straight line drawn parallel to the section's diameter from the cone's vertex to the [axial] triangle's base to the [rectangular] plane under the straight lines cut [on the axial triangle's base] by this straight line in the direction of the sides of the [axial] triangle, and the applied plane has as breadth the straight line on the diameter from the section's vertex to [the point] where the diameter is cut off by the straight line drawn from the section to the diameter, this plane is [the rectangular plane under two mentioned straight lines] and decreased by a figure similar and similarly situated to the plane under the mentioned straight line and the diameter. I will call such a section an ellipse.

Let there be a cone whose vertex is the point A and whose base is the circle ΒΓ, and let it be cut by a plane through its axis, and let it make as a section the triangle ΑΒΓ. And let it also be cut by another plane on the one hand meeting both [lateral] sides of the axial triangle and on the other hand continued neither parallel to the base of the cone, nor antiparallel to it, and let it make as a section on the surface of the cone the [closed curved] line ΔΕ. And let the common section of the cutting plane and of the plane of the base of the cone be ΖΗ perpendicular to ΒΓ, and let [according to Proposition I.7 and Definition 4] the diameter of the section be [the straight line] ΕΔ. And let ΕΘ be drawn from E perpendicular to [the diameter] ΕΔ, and let ΑΚ be drawn through A parallel to ΕΔ, and let it be contrived that as sq.ΑΚ is to pl.ΒΚΓ, so ΔΕ is to ΕΘ.

And let some point Λ be taken [at random] on the section, and let ΛΜ be drawn through Λ parallel to ΖΗ.

I say that ΛΜ is equal in square to the rectangular plane, which is applied to ΕΘ and having ΕΜ as breadth, and decreased by a figure similar to pl.ΔΕΘ.

[Proof]. For let ΔΘ be joined, and on the one hand let ΜΞΝ be drawn through M parallel to ΘΕ, and on the other hand let ΘΝ and ΞΟ be drawn through Θ and Ξ parallel to ΕΜ, and let ΠΜΡ be drawn through M parallel to ΒΓ

Since then ΠΡ is parallel to ΒΓ, and ΛΜ is also parallel to ΖΗ, therefore 18 [according to Proposition XI.15 of Euclid] the plane through ΛΜ and ΠΡ is parallel to the plane through ΖΗ and ΒΓ, which is to the base of the cone.

If therefore a plane is drawn through ΛΜ and ΠΡ, the section [according to Proposition I.4] will be a circle whose diameter is ΠΡ. And ΛΜ is perpendicular to it. Therefore [according to Proposition II.14 of Euclid] pl.ΠΜΡ is equal to sq.ΛΜ.

And since as sq.ΑΚ is to pl.ΒΚΓ, so ΕΔ is to ΕΘ, and [according to Proposition VI.23 of Euclid] the ratio sq.ΑΚ to pl.ΒΚΓ is compounded of [the ratios] ΑΚ to ΚΒ and ΑΚ to ΚΓ.

But [according to Proposition VI.4 of Euclid] as ΑΚ is to ΚΒ, so ΕΗ is to ΗΒ and ΕΜ is to ΜΠ, and as ΑΚ is to ΚΓ, so ΔΗ is to ΗΓ and ΔΜ is to ΜΡ.

Therefore the ratio ΔΕ to ΕΘ is compounded of the [ratios] ΕΜ to ΜΠ and ΔΜ to ΜΡ.

But [according to Proposition VI.23 of Euclid] the ratio pl.EMΔ to pl.ΠMP is compounded of the [ratios] EM to MΠ and ΔM to MP.

Therefore [according to Proposition VI.4 of Euclid] as pl.EMΔ is to pl.ΠMP, so ΔE is to EΘ and ΔM is to MΞ.

And with the straight line ME taken as common height [according to Proposition VI.1 of Euclid] as ΔM is to MΞ, so pl.ΔME is to pl.ΞME.

Therefore also [according to Proposition V.11 of Euclid] as pl.ΔME is to pl.ΠMP, so pl.ΔME is to pl.ΞME. Therefore [according to Proposition V.9 of Euclid] pl.ΠMP is equal to pl.ΞME.

But it was shown that pl.ΠMP is equal to sq.ΛM, therefore also pl.ΞME is equal to sq.ΛM.

Therefore ΛM is equal in square to the parallelogram MO, which is applied to ΘE and having EM as breadth and decreased by the figure ON similar to pl.ΔEΘ.

I will call such a section an ellipse, and let EΘ be called the straight line of application [of rectangular planes] to which the ordinates drawn to ΔE are equal in square. I will call this straight line also the latus rectum, and the straight line EΔ the latus transversum.